MTH202 Discrete Mathematics
Lecture Wise Questions and Answers
For Final Term Exam Preparation by Virtualians Social Network

Lecture No. 23.

Q. What is inductive step?

Ans.

In basis step we check that the given statement or proposition is true for n=1 and in Inductive step we first suppose that the proposition is true for n=k and prove that it is also true for n=k+1.

In the inductive step we suppose that a function is true for an arbitrary integer k and then we prove that the given statement is also hold for k+1. It means that the statement is true for all positive integers n

Q. How can we define propositional function ?

Ans.

In simple words propositional function is an expression in form of proposition but having undefined symbols (variables) and become a proposition by assigning appropriate values to these symbols.

Q. In the Basis step of mathematical Induction, I've seen in some problems P(0) is used while in some P(1) is used.

In the basic definition we've to find P(1) is true. So what's the reason of using P(0)?

Ans.

Principle of Mathematical Induction is defined as follow:

Let P(n) be a property that is defined for integers n, and let ' a ' be a fixed integer.

Suppose the following two statements are true:

1. P(a) is true.

2. For all integers $k \geq a$, if P(k) is true then P(k + 1) is true.

Then the statement for all integers $n \geq a$, P(n) is true.

Here P(1) is not fixed, we take P(a) for some fixed ' a ' and than suppose for some $k \geq a$, and so on.

Q. Explain the last exercise of the lecture (DeMorgan's Law)

Ans.

That is the DeMorgan's Law for the n sets. For understanding it, you should see first DeMorgan's Law for two sets A and B which is following:

$(A \cup B)^c = A^c \cap B^c$

For n number of sets A1, A2, ..., An, DeMorgan's Law is the following:

$(A_1 \cup A_2 \cup ... \cup A_n)^c = A_1^c \cap A_2^c \cap .... \cap A_n^c$

This is to be proved using Mathematical Induction in the lecture.
Q.In first example basic step why it is written that series is ending at one while interger are increasing?
Ans.
In the Mathematical Induction, we take some initial value of n in the basic step. This initial value can be 0 or 1.

In the example, we take n = 1 in the series. It means we take only one term in the series as an initial value for our Mathematical Induction.
Lecture No.24.
Q.what is consecutive positive integer and inequality?
Consecutive integers are integers that follow each other in order.
For Example
1,2,3,... are called consecutive positive integers. You can see, we will get next integer by adding 1 in the current integer.
Inequality is a statement in which equal sign is not involved or which is not equal.
For example:
$2 < 5$ or $7 > 4$.
Lecture No. 25
Q. What is the rule of direct proof?
In direct proof, the conclusion is established by logically combining the axioms, definitions, and earlier theorems. For example, direct proof can be used to establish that the sum of two even integers is always even.
Q. What is rational number and integer?
The natural numbers including 0 (0, 1, 2, 3, ...) together with the negatives of the non-zero natural numbers (−1, −2, −3, ...). they are numbers that can be written without a fractional or decimal component, and fall within the set {... −2, −1, 0, 1, 2, ...}. For example, 65, 7, and −756 are integers; 1.6 and 1½ are not integers.

A rational number is a number that can be in the form p/q where p and q are integers and q is not equal to zero.
Any whole number is rational. Its denominator is 1. For instance, 8 equals 8/1, which is the quotient of two integers. A number like square root of 16 is rational, since it can be expressed as

the quotient of two integers in the form 4/1. The following are also examples of rational numbers: 5/6,-6/7,3/1 etc

Q. Define Prime number?

A prime number is a whole number greater than 1, whose only two whole-number factors are 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. Diviser is the number that can divide your desired number. e.g. 6 has the following divisor 1,2,3,6.

Lecture No.26

Q. What is Direct Method and Indirect method?

There are two methods of proof Direct Method and Indirect Methods.

In Direct Method we try to prove the statement directly as it is .

For example "Prove that the sum of two odd integers is even." We will take two odd integers and try to prove that their sum is even.

While in Indirect Methods we try to restate the given statement and use this to prove our objective.

Q. Define Geometric Sequence or Geometric Progression (G.P.)?

A sequence in which every term after the first is obtained from the preceding term by multiplying it with a constant number is called a geometric sequence or geometric progression (G.P.)

The constant number, being the ratio of any two consecutive terms is called the common ratio of the G.P. commonly denoted by “r”.

Example: 6, -2, 2/3, - 2/9, … (common ratio = r = -1/3).

The sum of the terms of a geometric sequence forms a geometric series (G.S.). In general, if a is the first term and r the common ratio of a geometric series, then the series is given as: $a + ar + ar^2 + ar^3 + \dots$

For Example: 6 -2 + 2/3 - 2/9 +… (common ratio = r = -1/3)

is geometric series.

For finite series the formula is

$S_n= (a(1-r^n) /(1-r))$ since r ¹1.

For the given example: 6 -2 + 2/3 - 2/9 +… (common ratio = r = -1/3) is an infinite geometric series so the formula to find sum is

$S_n= (a/(1-r))$ since $|r| < 1$.

$S_n= (a/(r-1))$ if $|r| > 1$.

Lecture No. 27

Q. Define RECURSION?

It is the process of describing an action in terms of itself. OR

The process of defining an object in terms of smaller versions of itself is called recursion.

A recursive definition has two parts:

1.BASE:

An initial simple definition which cannot be expressed in terms of smaller versions of itself.

2. RECURSION:

The part of definition which can be expressed in terms of smaller versions of itself

Q.PRE-CONDITIONS AND POST-CONDITIONS?
Pre-condition is a condition or predicate that must always be true just prior to the theorems, definitions or before an operation in a formal specification.
Post-condition is a condition that must always be tested just after the Pre-condition.
Lecture No.28
Q. The loop invariant?
The loop invariant is a method which is used to prove the correctness of the loop with respect to certain per and post conditions. It is based on the principle of mathematical induction
Q. Division Algorithm?

For any integer n and a positive integer d, there exist unique integers q and r such that n = d • q + r and 0 ≤ r < d.

For example, for integers 5 and 3, there exist 1 and 2 such that 5 = 3x1 + 2

Q. The Euclidean Algorithm?

The Euclidean algorithm takes integers a and b with a > b ≥ 0 and calculate their greatest common divisor.

See the example at page #205

Q. Lemma?

Lemma is a proven result or proposition that helps for drawing the larger results.

Q. Euclidean algorithm?
In mathematics, the Euclidean algorithm (also called Euclid's algorithm) is an efficient method for computing the greatest common divisor (GCD) of two integers, also known as the greatest common factor (GCF) or highest common factor (HCF). It is named after the Greek mathematician Euclid.
The Euclidean algorithm is an algorithm for finding the greatest common divisor (G.C.D) of two numbers a and b.
Example: Suppose that a = 2322, b = 654.
2322 = 654•3 + 360
654 = 360•1 + 294
360 = 294•1 + 66
294 = 66•4 + 30
66 = 30•2 + 6
30 = 6•5
So gcd(2322,654) = 6

Lecture No.29

Q .What is combinatores?

Combinatory is the mathematics of counting and arranging objects.Counting of objects with certain properties (enumeration) is required to solve many different types of problem .For example,counting is used to:

(i) Determine number of ordered or unordered arrangement of objects.
(ii)Generate all the arrangements of a specified kind which is important in computer simulations.
(iii)Compute probabilities of events.
(iv)Analyze the chance of winning games, lotteries etc.
(v)Determine the complexity of algorithms

RULES FOR SUM

If one event can occur in n_1 ways, a second event can occur in n_2 (different) ways, then the total number of ways in which exactly one of the events (i.e., first or second) can occur is $n_1 + n_2$.

EXAMPLE:

Suppose there are 7 different optional courses in Computer Science and 3 different optional courses in Mathematics. Then there are 7 + 3 = 10
choices for a student who wants to take one optional course

EXERCISE:

A student can choose a computer project from one of the three lists. The three lists contain 23, 15 and 19 possible projects, respectively. How many possible projects are there to choose from?

SOLUTION:

The student can choose a project from the first list in 23 ways, from the second list in 15 ways, and from the third list in 19 ways. Hence, there are
23 + 15 + 19 = 57 projects to choose from

PRDUCT RULE IN TRMS OF SET

If A_1, A_2, …, A_m are finite sets, then the number of elements in the Cartesian product of these sets is the product of the number of elements in each set.

If $n(A_i)$ denotes the number of elements in set A_i, then
n(A_1 ´ A_2´ … ´A_m) = n (A_1) · n(A_2) ·… ·n ($A_{m,}$)

EXERCISE:

Find the number n of ways that an organization consisting of 15 members can elect a president, treasurer, and secretary. (assuming no person is elected to more than one position)

SOLUTION:

The president can be elected in 15 different ways; following this, the treasurer can be elected in 14 different ways; and following this, the secretary can be elected in 13 different ways. Thus, by product rule, there are
$n = 15 \times 14 \times 13 = 2730$
different ways in which the organization can elect the officers

Lecture No.30

Q.FACTORIAL OF A POSITIVE INTEGER?

For each positive integer n, its factorial is defined to be the product of all the integers from 1 to n and is denoted n!. Thus n! = n(n - 1) (n - 2) … 3×2 ×1

In addition, we define

0! = 1

REMARK:

n! can be recursively defined as

Base: 0! = 1

Recursion n! = n (n - 1)! for each positive integer N

Lecture No.30

FACTORIAL OF A POSITIVE INTEGER:

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REMARK:

n! can be recursively defined as

Base: 0! = 1

Recursion n! = n (n - 1)! for each positive integer N

Lecture No.31

A *k*-combination of a set of *n* elements is a choice of *k* elements taken from the set of *n* elements such that the order of the elements does not matter and elements can't be repeated.

The symbol *C*(*n*, *k*) denotes the number of *k*-combinations that can be chosen from a set of *n* elements.

EXAMPLE:

Let *X* = {*a*, *b*, *c*}. Then 2-combinations of the 3 elements of the set *X* are:

{*a*, *b*}, {*a*, *c*}, and {*b*, *c*}. Hence *C*(3,2) = 3.

EXERCISE:

Let *X* = {*a*, *b*, *c*, *d*, *e*}.

List all 3-combinations of the 5 elements of the set *X*, and hence find the value of *C*(5,3).

SOLUTION:

Then 3-combinations of the 5 elements of the set *X* are:

{*a*, *b*, *c*}, {*a*, *b*, *d*}, {*a*, *b*, *e*}, {*a*, *c*, *d*}, {*a*, *c*, *e*},

{*a*, *d*, *e*}, {*b*, *c*, *d*}, {*b*, *c*, *e*}, {*b*, *d*, *e*}, {*c*, *d*, *e*}

Lecture No.32

DEFINITION:

A *k*-selection of a set of *n* elements is a choice of *k* elements taken from a set of *n* elements such that the order of elements does not matter and elements can be repeated.

REMARK:

1. *k*-selections are also called *k*-combinations with repetition allowed or multisets of size *k*.
2. With *k*-selections of a set of *n* elements repetition of elements is allowed. Therefore *k* need not to be less than or equal to *n*.

ORDERED AND UNORDERED PARTITIONS:

An unordered partition of a finite set S is a collection [A_1, A_2, …, A_k] of disjoint (nonempty) subsets of S (called cells) whose union is S.

The partition is ordered if the order of the cells in the list counts.

EXAMPLE:

Let S = {1, 2, 3, …, 7}

The collections

P_1 = [{1,2}, {3,4,5}, {6,7}]

And P_2 = [{6,7}, {3,4,5}, {1,2}]

determine the same partition of S but are distinct ordered partitions

Lecture No.34

PIGEONHOLE PRINCIPLE:

A function from a set of $k + 1$ or more elements to a set of k elements must have at least two elements in the domain that have the same image in the co-domain.

If $k + 1$ or more pigeons fly into k pigeonholes then at least one pigeonhole must contain two or more pigeons.

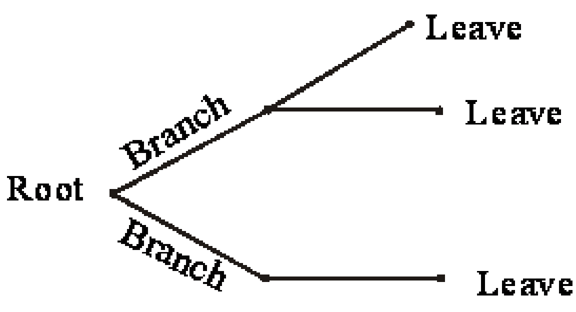
Lecture No.35

TREE DIAGRAM

A tree diagram is a useful tool to list all the logical possibilities of a sequence of events where each event can occur in a finite number of ways.

A tree consists of a root, a number of branches leaving the root, and possible additional branches leaving the end points of other branches. To use trees in counting problems, we use a branch to represent each possible choice. The possible outcomes are represented by the leaves (end points of branches).

A tree is normally constructed from left to right.



A TREE STRUCTURE

Q. Definition of tree diagram?

graphical tool which systematically breaks down, and then maps out in increasing detail, all components or elements of a condition phenomenon, process , or situation, at successive levels or stages . In case of a 'divergent tree,' it begins with a single entry that has one or more paths (branches) leading out from it, some or all of which subdivide into more branches. This process is repeated until all possibilities are exhausted. In case of a 'convergent tree,' this process works in reverse. family(genealogical) and organization charts are the common examples of a tree diagram. Also called chain of causes or dendrite diagram.

Lecture No.37

Q. Conditional probability?

CONDITIONAL PROBABILITY
MULTIPLICATION THEOREM
INDEPENDENT EVENTS

EXAMPLE:

a. What is the probability of getting a 2 when a dice is tossed?

b. An even number appears on tossing a die.

(i) What is the probability that the number is 2?

(ii) What is the probability that the number is 3?

SOLUTION:

When a dice is tossed, the sample space is S={1,2,3,4,5,6} ? ? ? ? ? n(S)=6

a. Let "A" denote the event of getting a 2 i.e A={ 2 } ? ? n(A)=1

$$P(\text{2 appears when the die is tossed}) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

b. (i) Let "S_1" denote the total number of even numbers from a sample space S,when a dice is tossed (i.e $S_1 \subseteq S$)

S_1={ 2,4,6 } $n(S_1)=3$

Let "B" denote the event of getting a 2 from total number of even number i.e B={ 2 }

? n(B) = 1

$$P(\text{2 appears; given that the number is even})=P(B)= \frac{n(B)}{n(S_1)} = \frac{1}{3}$$

(ii) Let "C" denote the event of getting a 3 in S_1(among the even numbers)i.e C={ }

n(C)=0

$$P(\text{3 appears; given that the number is even}) = P(C) = \frac{n(C)}{n(S_1)} = \frac{0}{3} = 0$$

EXAMPLE:

Suppose that an urn contains 3 red balls, 2 blue balls, and 4 white balls, and that a ball is selected at random.

Let E be the event that the ball selected is red.

Then P(E) = 3/9 (as there are 3 red balls out of total 9 balls)

Let F be the event that the ball selected is not white.

Then the probability of E if it is already known that the selected ball is not white would be

P(red ball selected; given that the selected ball is not white) = 3/5 (as we count no white ball so there are total 9 balls(i.e 2 blue and 3 red balls))

This is called the conditional probability of E given F and is denoted by P(E|F).

DEFINITION:

Let E and F be two events in the sample space of an experiment with $P(F) \neq 0$. The **conditional probability** of E given F, denoted by $P(E|F)$, is defined as

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

EXAMPLE:

Let A and B be events of an experiment such that
$P(B) = 1/4$ and $P(A \cap B) = 1/6$.
What is the conditional probability $P(A|B)$?

SOLUTION:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/4} = \frac{4}{6} = \frac{2}{3}$$

EXERCISE:

Let A and B be events with $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$

Find

(i) $P(A|B)$ (ii) $P(B|A)$

(iii) $P(A \cup B)$ (iv) $P(A^c | B^c)$

SOLUTION:

Using the formula of the conditional Probability we can write

(i) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{1/4}{1/3} = \frac{3}{4}$$

(ii) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$$ (As $P(B \cap A) = P(A \cap B) = 1/4$)

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{7}{12}$$

Lecture No.38

RANDOM VARIABLE

PROBABILITY DISTRIBUTION

EXPECTATION AND VARIANCE

INTRODUCTION:

Suppose S is the sample space of some experiment. The outcomes of the experiment, or the points in S, need not be numbers. For example in tossing a coin, the outcomes are H (heads) or T(tails), and in tossing a pair of dice the outcomes are pairs of integers. However, we frequently wish to assign a specific number to each outcome of the experiment. For example, in coin tossing, it may be convenient to assign 1 to H and 0 to T; or in the tossing of a pair of dice, we may want to assign the sum of the two integers to the outcome. Such an assignment of numerical values is called a random variable.

RANDOM VARIABLE:

A random variable X is a rule that assigns a numerical value to each outcome in a sample Space S.

OR

It is a function which maps each outcome of the sample space into the set of real numbers.

We shall let X(S) denote the set of numbers assigned by a random variable X, and refer to X(S) as the range space.

In formal terminology, X is a function from S(sample space) to the set of real numbers R, and X(S) is the range of X.

REMARK:

1. A random variable is also called a chance variable, or a stochastic variable(not called simply a variable, because it is a function).

2. Random variables are usually denoted by capital letters such as X, Y, Z; and the values taken by them are represented by the corresponding small letters.

EXAMPLE:

A pair of fair dice is tossed. The sample space S consists of the 36 ordered pairs i.e

$S = \{(1,1), (1,2), (1,3), \ldots, (6,6)\}$

Let X assign to each point in S the sum of the numbers; then X is a random variable with range space i.e

$X(S) = \{2,3,4,5,6,7,8,9,10,11,12\}$

Let Y assign to each point in S the maximum of the two numbers in the outcomes; then Y is a random variable with range space.

$Y(S) = \{1,2,3,4,5,6\}$

PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE:

Let $X(S) = \{x_1, x_2, \ldots, x_n\}$ be the range space of a random variable X defined on a finite sample space S.

Define a function f on X(S) as follows:

$f(x_i) = P(X = x_i)$

= sum of probabilities of points in S whose image is x_i.
This function f is called the probability distribution or the probability function of X.
The probability distribution f of X is usually given in the form of a table.

x_1	x_2	…	x_n
$f(x_1)$	$f(x_2)$	…	$f(x_n)$

The distribution f satisfies the conditions.

$$(i) \quad f(x_i) \geq 0 \quad \text{and} \quad (ii) \quad \sum_{i=1}^{n} f(x_i) = 1$$

Q. Definition of random variable?
A random variable is a variable whose value is subject to variations due to chance (i.e. randomness, in a mathematical sense). As opposed to other mathematical variables, a random variable conceptually does not have a single, fixed value (even if unknown); rather, it can take on a set of possible different values, each with an associated probability.
A random variable's possible values might represent the possible outcomes of a yet-to-be-performed experiment or an event that has not happened yet, or the potential values of a

past experiment or event whose already-existing value is uncertain (e.g. as a result of incomplete information or imprecise measurements).

They may also conceptually represent either the results of an "objectively" random process (e.g. rolling a die), or the "subjective" randomness that results from incomplete knowledge of a quantity. The meaning of the probabilities assigned to the potential values of a random variable is not part of probability theory itself, but instead related to philosophical arguments over the interpretation of probability. The mathematics works the same regardless of the particular interpretation in use.

Lecture No.39

Q. Graph Theory?

Graph theory plays an important role in several areas of computer science such as:

- switching theory and logical design
- Artificial intelligence
- Formal languages
- Computer graphics
- Operating systems
- Compiler writing
- Information organization and retrieval.

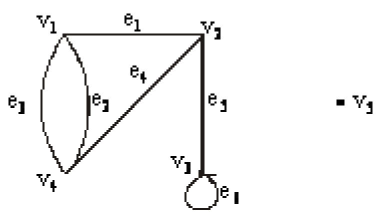
GRAPH

A graph is a non-empty set of points called vertices and a set of line segments joining pairs of vertices called edges.

Formally, a graph G consists of two finite sets:

(i) A set V=V(G) of vertices (or points or nodes)

(ii)A set E=E(G) of edges; where each edge corresponds to a pair of vertices.



The graph G with

$V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and

$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

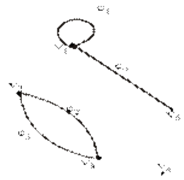
DRAWING PICTURE FOR A GRAPH

Draw picture of Graph H having vertex set $\{v_1, v_2, v_3, v_4, v_5\}$ and edge set $\{e_1, e_2, e_3, e_4\}$ with edge endpoint function

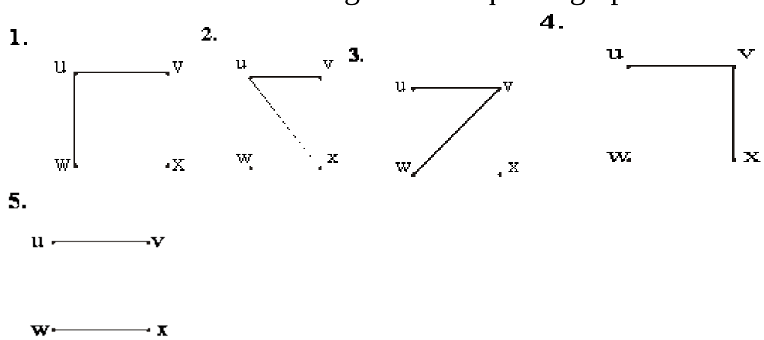
Edge	Endpoint
e_1	$\{v_1\}$
e_2	$\{v_2,v_3\}$
e_3	$\{v_2,v_3\}$

e_4	$\{v_1,v_5\}$

SOLUTION:
Given $V(H) = \{v_1, v_2, v_3, v_4, v_5\}$
and $E(H) = \{e_1, e_2, e_3, e_4\}$
with edge endpoint function



SOLUTION:
There are C(4,2) = 6 ways of choosing two vertices from 4 vertices. These edges may be listed as:
{u,v},{u,w},{u,x},{v,w}, {v,x},{w,x}
One edge of the graph is specified to be {u,v}, so any of the remaining five from this list may be chosen to be the second edge. This required graphs are:



Lecture No.40
PATHS AND CIRCUITS
KONIGSBERG BRIDGES PROBLEM
DEFINITIONS:
Let G be a graph and let v and w be vertices in graph G.
1. WALK
A walk from v to w is a finite alternating sequence of adjacent vertices and edges of G.
Thus a walk has the form
$v_0e_1v_1e_2 \dots v_{n-1}e_nv_n$
where the v's represent vertices, the e's represent edges $v_0=v$, $v_n=w$, and for all
i = 1, 2 … n, v_{i-1} and v_i are endpoints of e_i.
The trivial walk from v to v consists of the single vertex v.
2. CLOSED WALK
A closed walk is a walk that starts and ends at the same vertex.
3. CIRCUIT
A circuit is a closed walk that does not contain a repeated edge. Thus a circuit is a walk of the form

$v_0e_1v_1e_2 \ldots v_{n-1}\ e_n\ v_n$
where $v_0 = v_n$ and all the e_i,s are distinct.
Q.SIMPLE CIRCUIT?
A simple circuit is a circuit that does not have any other repeated vertex except the first and last.
Thus a simple circuit is a walk of the form
$v_0e_1v_1e_2 \ldots v_{n-1}e_nv_n$
where all the e_i,s are distinct and all the v_j,s are distinct except that $v_0 = v_n$
5. PATH
A path from v to w is a walk from v to w that does not contain a repeated edge.
Thus a path from v to w is a walk of the form
$v = v_0e_1v_1e_2 \ldots v_{n-1}e_nv_n = w$
where all the e_i,s are distinct (that is e_i ¹ e_k for any i ¹k).
6. SIMPLE PATH
A simple path from v to w is a path that does not contain a repeated vertex.
Thus a simple path is a walk of the form
$v = v_0e_1v_1e_2 \ldots v_{n-1}e_nv_n= w$
where all the e_i,s are distinct and all the v_j,s are also distinct (that is, v_j ¹ v_m for any
j ¹ m).

Q. EULER PATH?

DEFINITION:

Let G be a graph and let v and w be two vertices of G. An Euler path from v to w is a sequence of adjacent edges and vertices that starts at v, end and w, passes through every vertex of G at least once, and traverses every edge of G exactly once.

Q. COROLLARY?

Let G be a graph and let v and w be two vertices of G. There is an Euler path from v to w if, and only if, G is connected, v and w have odd degree and all other vertices of G have even degree.

Q. HAMILTONIAN CIRCUITS?

DEFINITION:

Given a graph G, a Hamiltonian circuit for G is a simple circuit that includes every vertex of G. That is, a Hamiltonian circuit for G is a sequence of adjacent vertices and distinct edges in which every vertex of G appears exactly once.

Lecture No.41
Q. Matrix Representation of Graphs?
Definitions:
In this section, we introduce two kinds of matrix representations of a graph,
that is, the adjacency matrix and incidence matrix of the graph.
A graph G with the vertex-set V (G) = {x1, x2, · · · , vv} can be described by
means of matrices. The adjacency matrix of G is a v × v matrix
A(G) = (aij), where aij = μ(xi, xj) = |EG(xi, xj)|.

The adjacency matrix or the incidence matrix of a graph is another representation of the graph, and it is this form that a graph can be commonly stored in computers. The matrix representation of a graph is often convenient if one intends to use a computer to obtain some information or solve a problem concerning the graph. This kind of representation of a graph is conducive to study properties of the graph by means of algebraic methods.

It is not difficult to see that the adjacency matrices of two isomorphic graphs are per mutedly similar. In other words, assume that A and B are the adjacency matrices of two isomorphic graphs G and H, respectively, then there exists a v × v permutation matrix P such that A = P−1BP.

Similarly, the incidence matrices of two isomorphic graphs are per mutedly equivalent. In other words, assume that M and N are the incidence matrices of two isomorphic graphs G and H, respectively, then there exist a v ×v permutation matrix P and an " × " permutation matrix Q such that M = PNQ.

Q. Adjacency matrix?

An adjacency matrix is a means of representing which vertices (or nodes) of a graph are adjacent to which other vertices. Another matrix representation for a graph is the incidence matrix. Specifically, the adjacency matrix of a finite graph G on n vertices is the n × n matrix where the non-diagonal entry aij is the number of edges from vertex i to vertex j, and the diagonal entry aii, depending on the convention, is either once or twice the number of edges (loops) from vertex i to itself. Undirected graphs often use the latter convention of counting loops twice, whereas directed graphs typically use the former convention. There exists a unique adjacency matrix for each isomorphism class of graphs (up to permuting rows and columns), and it is not the adjacency matrix of any other isomorphism class of graphs. In the special case of a finite simple graph, the adjacency matrix is a (0,1)-matrix with zeros on its diagonal. If the graph is undirected, the adjacency matrix is symmetric.

The relationship between a graph and the eigenvalues and eigen vectors of its adjacency matrix is studied in spectral graph theory

Lecture No.42

ISOMORPHISM OF GRAPHs

An isomorphism of graphs *G* and *H* is a bijection between the vertex sets of *G* and *H* such that any two vertices *u* and *v* of *G* are adjacent in *G* if and only if $f(u)$ and $f(v)$ are adjacent in *H*. This kind of bijection is commonly called "edge-preserving bijection", in accordance with the general notion of isomorphism being a structure-preserving bijection.

In the above definition, graphs are understood to be undirected non-labeled nonweighted graphs. However, the notion of isomorphism may be applied to all other variants of the notion of graph, by adding the requirements to preserve the corresponding additional elements of structure: arc directions, edge weights, etc., with the following exception. When spoken about graph labeling with *unique labels,* commonly taken from the integer range 1,...,*n*, where *n* is the number of the vertices of the graph, two labeled graphs are said to be isomorphic if the corresponding underlying unlabeled graphs are isomorphic.

If an isomorphism exists between two graphs, then the graphs are called isomorphic and we write . In the case when the bijection is a mapping of a graph onto itself, i.e., when *G* and *H* are one and the same graph, the bijection is called an automorphism of *G*.

The graph isomorphism is an equivalence relation on graphs and as such it partitions the class of all graphs into equivalence classes. A set of graphs isomorphic to each other is called an isomorphism class of graphs

ISOMORPHIC GRAPHS:

Let G and G’ be graphs with vertex sets V(G) and V(G’) and edge sets E(G) and E(G’), respectively.

G is isomorphic to G’ if, and only if, there exist one-to-one correspondences g: V(G)®V(G’) and h: E(G) ®E(G’) that preserve the edge-endpoint functions of G and G’ in the sense that for all v ÎV(G) and eÎ E(G).

v is an endpoint of e Û g(v) is an endpoint of h(e).

EQUIVALENCE RELATION:

Graph isomorphism is an equivalence relation on the set of graphs.

1. Graphs isomorphism is Reflexive (It means that the graph should be isomorphic to itself).
2. Graphs isomorphism is Symmetric (It means that if G is isomorphic to G’ then G’ is also isomorphic to G).
3. Graphs isomorphism is Transitive (It means that if G is isomorphic to G’ and G’ is isomorphic to G’’, then G is isomorphic to G’’).

ISOMORPHIC INVARIANT:

A property P is called an isomorphic invariant if, and only if, given any graphs G and G’, if G has property P and G’ is isomorphic to G, then G’ has property P.

THEOREM OF ISOMORPHIC INVARIANT:

Each of the following properties is an invariant for graph isomorphism, where n, m and k are all non-negative integers, if the graph:

1. has n vertices.
2. has m edges.
3. has a vertex of degree k.
4. has m vertices of degree k.
5. has a circuit of length k.
6. has a simple circuit of length k.
7. has m simple circuits of length k.
8. is connected.
9. has an Euler circuit.
10. has a Hamiltonian circuit.

DEGREE SEQUENCE:

The degree sequence of a graph is the list of the degrees of its vertices in non-increasing order.

GRAPH ISOMORPHISM FOR SIMPLE GRAPHS:

If G and G' are simple graphs (means the "graphs which have no loops or parallel edges") then G is isomorphic to G' if, and only if, there exists a one-to-one correspondence (1-1 and onto function) g from the vertex set V (G) of G to the vertex set
V (G') of G' that preserves the edge-endpoint functions of G and G' in the sense that for all vertices u and v of G,
{u, v} is an edge in G Û {g(u), g(v)} is an edge in G'.
OR
You can say that with the property of one-one correspondence, u and v are adjacent in graph G Û if g (u) and g (v) are adjacent in G'.
Note:
It should be noted that unfortunately, there is no efficient method for checking that whether two graphs are isomorphic(methods are there but take so much time in calculations).Despite that there is a simple condition. Two graphs are isomorphic if they have the same number of vertices(as there is a 1-1 correspondence between the vertices of both the graphs) and the same number of edges(also vertices should have the same degree.
Q.Degree sequence?
The degree sequence of an undirected graph is the non-increasing sequence of its vertex degrees. The degree sequence is a graph invariant so isomorphic graphs have the same degree sequence. However, the degree sequence does not, in general, uniquely identify a graph; in some cases, non-isomorphic graphs have the same degree sequence.
The degree sequence problem is the problem of finding some or all graphs with the degree sequence being a given non-increasing sequence of positive integers. (Trailing zeroes may be ignored since they are trivially realized by adding an appropriate number of isolated vertices to the graph.) A sequence which is the degree sequence of some graph, i.e. for which the degree sequence problem has a solution, is called a graphic or graphical sequence. As a consequence of the degree sum formula, any sequence with an odd sum, such as (3, 3, 1), cannot be realized as the degree sequence of a graph. The converse is also true: if a sequence has an even sum, it is the degree sequence of a multigraph.
Lecture No.43
PLANAR GRAPHS
GRAPH COLORING
Graph coloring is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color; this is called a vertex coloring. Similarly, an edge coloring assigns a color to each edge so that no two adjacent edges share the same color, and a face coloring of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.
Vertex coloring is the starting point of the subject, and other coloring problems can be transformed into a vertex version. For example, an edge coloring of a graph is just a vertex coloring of its line graph and a face coloring of a plane graph is just a vertex coloring of its dual However, non-vertex coloring problems are often stated and studied *as is*. That is partly for

perspective, and partly because some problems are best studied in non-vertex form, as for instance is edge coloring.

The convention of using colors originates from coloring the countries of a map, where each face is literally colored. This was generalized to coloring the faces of a graph embedded in the plane. By planar duality it became coloring the vertices, and in this form it generalizes to all graphs. In mathematical and computer representations, it is typical to use the first few positive or nonnegative integers as the "colors". In general, one can use any finite set as the "color set". The nature of the coloring problem depends on the number of colors but not on what they are.

In this graph, note it that each vertex is connected to every other vertex, but no edge is crossed.

Note: The graphs shown above are complete graphs with four vertices (denoted by K_4).

DEFINITION:

A graph is called planar if it can be drawn in the plane without any edge crossed (crossing means the intersection of lines). Such a drawing is called a plane drawing of the graph.

OR

You can say that a graph is called planar in which the graph crossing number is “0”.

HOW TO DRAW A GRAPH FROM A MAP:

1. Each map in the plane can be represented by a graph.
2. Each region is represented by a vertex (in 1st map as there are 7 regions, so 7 vertices are used in drawing a graph, similarly we can see 2nd map).
3. If the regions connected by these vertices have the common border, then edge connect two vertices.
4. Two regions that touch at only one point are not adjacent.

DEFINITION:

1. A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.
2. The chromatic number of a graph is the least (minimum) number of colors for coloring of this graph

Lecture No.44

TREES

APPLICATION AREAS:

Trees are used to solve problems in a wide variety of disciplines. In computer science trees are employed to

1) construct efficient algorithms for locating items in a list.
2) construct networks with the least expensive set of telephone lines linking distributed computers.
3) construct efficient codes for storing and transmitting data.
4) model procedures that are carried out using a sequence of decisions, which are valuable in the study of sorting algorithms.

TREE:

A tree is a connected graph that does not contain any non-trivial circuit. (i.e. it is circuit-free).

A trivial circuit is one that consists of a single vertex.

SOME SPECIAL TREES

1. TRIVIAL TREE:

A graph that consists of a single vertex is called a trivial tree or degenerate tree.

2. EMPTY TREE

A tree that does not have any vertices or edges is called an empty tree.

3. FOREST

A graph is called a forest if, and only if, it is circuit-free.

OR "Any non-connected graph that contains no circuit is called a forest."

Hence, it clears that the connected components of a forest are trees.

PROPERTIES OF TREES:

1. A tree with n vertices has n - 1 edges (where n ≥0).
2. Any connected graph with n vertices and n - 1 edges is a tree.
3. A tree has no non-trivial circuit; but if one new edge (but no new vertex) is added to it, then the resulting graph has exactly one non-trivial circuit.
4. A tree is connected, but if any edge is deleted from it, then the resulting graph is not connected.
5. Any tree that has more than one vertex has at least two vertices of degree 1.
6. A graph is a tree iff there is a unique path between any two of its vertices

ROOTED TREE:

A rooted tree is a tree in which one vertex is distinguished from the others and is called the *root.*

The level of a vertex is the number of edges along the unique path between it and the root.

The heightof a rooted tree is the maximum level to any vertex of the tree.

The children of any internal vertex v are all those vertices that are adjacent to v and are one level farther away from the root than v.

If w is a child of v, then v is called the parent of w.

Two vertices that are both children of the same parent are called siblings.

Given vertices v and w, if v lies on the unique path between w and the root, then v is an ancestor of w and w is a descendant of v.

BINARY TREE

A *binary tree* is a rooted tree in which every internal vertex has at most two children.

Every child in a binary tree is designated either a left child or a right child (but not both).

A *full binary tree* is a binary tree in which each internal vertex has exactly two children.

Lecture No.45

SPANNING TREES:

SPANNING TREE:

A spanning tree for a graph G is a subgraph of G that contains every vertex of G and is a tree.

REMARK:

1. Every connected graph has a spanning tree.
2. A graph may have more than one spanning trees.
3. Any two spanning trees for a graph have the same number of edges.

4. If a graph is a tree, then its only spanning tree is itself.

MINIMAL SPANNING TREE:

A minimal spanning tree for a weighted graph is a spanning tree that has the least possible total weight compared to all other spanning trees of the graph.

If G is a weighted graph and e is an edge of G then w(e) denotes the weight of e and w(G) denotes the total weight of G.

KRUSKAL'S ALGORITHM:

Input: G [a weighted graph with n vertices]

Algorithm:

1. Initialize T(the minimal spanning tree of G) to have all the vertices of G and no edges.
2. Let E be the set of all edges of G and let m: = 0.
3. While (m < n - 1)

3a. Find an edge e in E of least weight.

3b. Delete e from E.

3c. If addition of e to the edge set of T does not produce a circuit then add e to the edge set of T and set m: = m + 1

end while

Output T

end Algorithm