Proude To Be Virtulian MOAAZ SIDDIQ	**MTH202- Discrete Mathematics** **LATEST SOLVED SUBJECTIVES** **FROM Final term PAPERS**	**July 12,2011**
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FINALTERM EXAMINATION
Spring 2011

Q1:Let $R \to R$ be defined by **5marks**

$$f(x) = \frac{2x+1}{2x+2}$$

Is f one-to-one?

Solution:

$$\frac{2x_1+1}{2x_1+2} = \frac{2x_2+1}{2x_2+2}$$

$$(2x_1+1)(2x_2+2) = (2x_2+1)(2x_1+2)$$

$$4x_1x_2 + 4x_1 + 2x_2 + 2 = 4x_1x_2 + 4x_2 + 2x_1 + 2$$

$$\cancel{4x_1x_2} + 4x_1 + 2x_2 \cancel{+2} - \cancel{4x_1x_2} - 4x_2 - 2x_1 \cancel{-2} = 0$$

$$4x_1 + 2x_2 - 4x_2 - 2x_1 = 0$$

$$4x_1 - 2x_1 = 4x_2 - 2x_2$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

$x_1 = x_2$ Therefore this function is one to one function

Q2: Find the gcd of 1075, 45 using dividing algorithm. **5marks**

Solution:

1.Divide 1075 by 45:
This gives 1075 = 45 * 23 + 40
2.Divide 45 by 40:
This gives 45 = 40 · 1 + 5
3.Divide 40 by 5:
This gives 40 = 5 * 8 + 0
Hence gcd (1075, 45) = 5.

Q 3. Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for x=-2.01 3 marks

Solution: Page 249

$\lfloor -2.01 \rfloor = \lfloor -3 + 0.99 \rfloor = -3$
$\lceil -2.01 \rceil = \lceil -3 + 0.999 \rceil = -3 + 1 = -2$

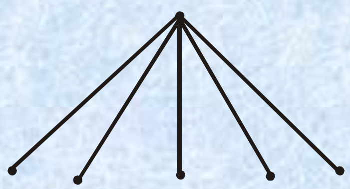
Q 4: Find the greatest common division for the following pair of integer: 30,105 2marks
Solution:

1.Divide 105 by 30:
This gives 105 = 30 * 5 +0
Hence gcd (105,30) = 30

Q 5: Find the Spanning tree for the graph $K_{1,5}$? 2marks

Solution: Page 332

$k_{1,5}$ represents a complete bipartite graph on (1,5) vertices, drawn below:



Clearly the graph itself is a tree (six vertices and five edges). Hence the graph is itself a spanning tree.

Q 6 Determine which f is a function?

$$f(x) = \frac{1}{n^2 - 4}$$

Q7. What is the difference between? {a,b} and {{a,b}}?
Solution:

{a,b} is a set while {{a,b}} is a subset of some set.

Q8. How many 3-digits can be formed by using each one of the digits 2,3,5,7,9 only once?

Solution:
5* 4* 3 = 60

Q9 what is the smallest integer N such that [N/9] =6?

$$N = 9\times\left(6-1\right)+ 1$$

$$= 9\times 5+1 = 46$$

FINALTERM EXAMINATION
Spring 2010
MTH202- Discrete Mathematics (Session - 2)

Question No: 31   (Marks: 2)
Let A and B be the events. Rewrite the following event using set notation
"A or not B occurs"

Question No: 32   (Marks: 2)
Find a non-isomorphic tree with four vertices.
Solution: Page 323

Any tree with four vertices has (4-1=3) three edges. Thus, the total degree of a tree with 4 vertices must be 6 [by using total degree=2(total number of edges)].

Also, every tree with more than one vertex has at least two vertices of degree 1, so the only possible combinations of degrees for the vertices of the trees are 1, 1, 1, 3 and 1, 1, 2, 2.
The corresponding trees (clearly non-isomorphic, by definition) are

Question No: 33   (Marks: 2)
Write the following in the factorial form:
n(n -1)(n-2)…(n-r+1)
Solution Page 217:

$$n\left(n-1\right)\left(n-2\right)\ldots\left(n-r+1\right) = \frac{n\left(n-1\right)\left(n-2\right)\ldots\left(n-r+1\right)-\left(n-r\right)!}{\left(n-r\right)!}$$

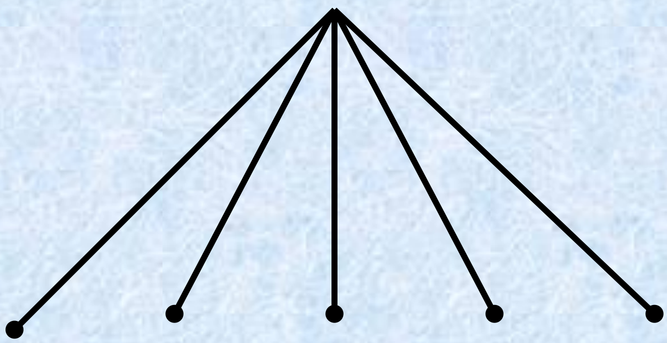
$$= \frac{n!}{\left(n-r\right)!}$$

Question No: 34   (Marks: 3)
Compute ëxû and éxù for x = 25/4

Question No: 35   (Marks: 3)
Find a spanning tree for the graph $k_{1,5}$?

$k_{1,5}$ represents a complete bipartite graph on (1,5) vertices, drawn below:

Solution (Page 332):



Clearly the graph itself is a tree (six vertices and five edges). Hence the graph is itself a spanning tree.

Question No: 36 (Marks: 3)
The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution:
$C(12,3) \times C(8,2) = 220 \times 28 = 6160$

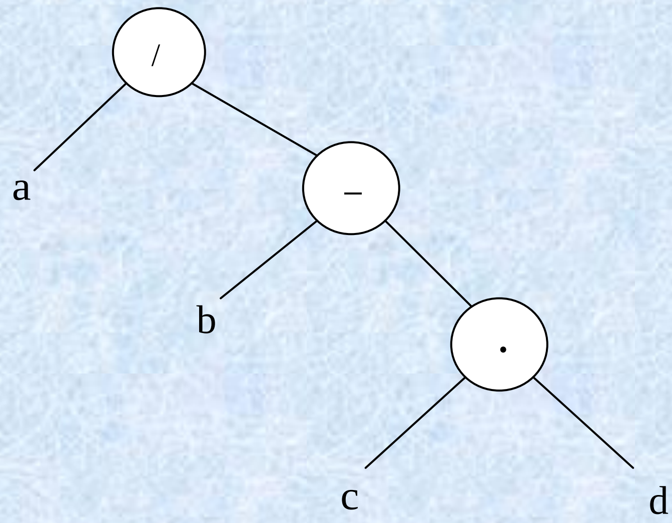
Question No: 37 (Marks: 5)
Is it possible to have a simple graph with four vertices of degree 1, 1, 3, and 3.If no then give reason?(Justify your answer)
Solution:

Yes, It is possible to make a graph with four vertices of degree 1, 1, 3, 3
Because 1+1+3+3=8
And According to handshaking theorem, the sum of the degrees of all the vertices of G equals twice the number of edges of G.
2*4=8
So it is possible.

Question No: 38 (Marks: 5)
Draw a binary tree to represent the following expression
a/(b-c.d)

The internal vertices are arithmetic operators, the terminal vertices are variables and the operator at each vertex acts on its left and right sub trees in left-right order.

Solution:



Question No: 39 (Marks: 5)
There are 25 people who work in an office together. Four of these people are selected to attend four different conferences. The first person selected will go to a conference in New York, the second will go to Chicago, the third to San Francisco, and the fourth to Miami. How many such selections are possible?

FINALTERM EXAMINATION
Spring 2010
MTH202- Discrete Mathematics (Session - 1)

Question No: 31 (Marks: 2)
Let A and B be the events. Rewrite the following event using set notation
"Only A occurs"
AB^c

Question No: 32 (Marks: 2)
Suppose that a connected planar simple graph has 15 edges. If a plane drawing of this graph has 7 faces, how many vertices does this graph have?

Answer:

Given,
Edges = v =15
Faces = f = 7

Vertices = v =?
According toEuler Formula, we know that,

$f= e – v +2$

Putting values, we get

$7 = 15 – v + 2$

$7 = 17 – v$

Simplifying

$v =1 7-7 =10$

Question No: 33 (Marks: 2)

How many ordered selections of two elements can be made from the set {0,1,2,3}?

Answer:

The order selection of two elements from 4 is as

P (4, 2) = 4! / (4-2)!

= (4.3.2.1)/2!

= 12

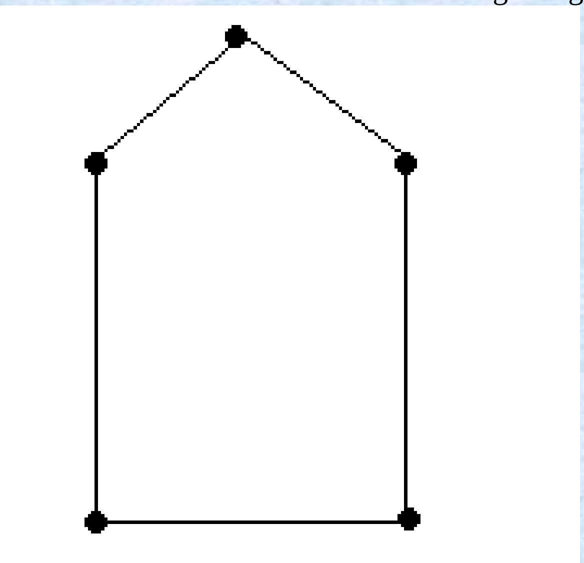
Question No: 34 (Marks: 3)

Consider the following events for a family with children:

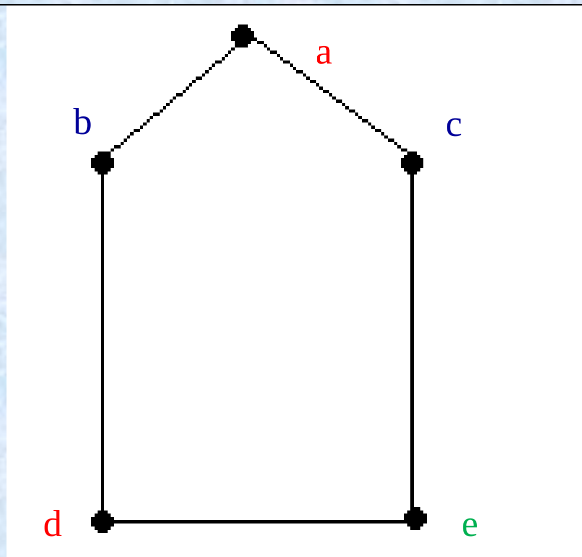
A={children of both sexes}, B={at most one boy}.Show that A and B are dependent events if a family has only two children.

Question No: 35 (Marks: 3)

Determine the chromatic number of the given graph by inspection.



Solution:



The **chromatic number** of a graph is the least (minimum) number of colors for coloring of this graph.
So chromatic number in this graph is 3

Question No: 36 (Marks: 3)
A cafeteria offers a choice of two soups, five sandwiches, three desserts and three drinks. How many different lunches, each consisting of a soup, a sandwich, a dessert and a drink are possible?
Solution:

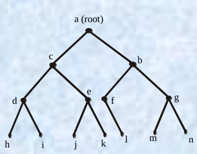
$$C(13,4) = \frac{13!}{4! \times (13-4)!}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times \not{9}!}{4! \times \not{9}!} = \frac{17160}{24} = 715$$

Question No: 37 (Marks: 5)
A box contains 15 items, 4 of which are defective and 11 are good. Two items are selected. What is probability that the first is good and the second defective?

Question No: 38 (Marks: 5)
Draw a binary tree with height 3 and having seven terminal vertices.
Solution: On Page 327

Given height=h=3
Any binary tree with height 3 has atmost 2^3=8 terminal vertices.
But here terminal vertices are 7
and Internal vertices=k=6 so binary tree exists and is as follows:



Question No: 39 (Marks: 5)
Find n if
P(n,2) = 72
Solution:

$P(n,2) = 72$

$n(n-1) = 72$ by using the definition of permutation

$n^2 - n = 72$

$n^2 - n - 72 = 0$

$n^2 + 8n - 9n - 72 = 0$

$n(n+8) - 9(n+8) = 0$

$(n-9)(n+8) = 0$

$n-9=0 \qquad n+8=0$

$n=9 \qquad n=-8$

n = 9 or -8

since n must be positive so only the acceptable value for n is 9

FINALTERM EXAMINATION
Fall 2009
MTH202- Discrete Mathematics

(Marks: 2)
Find the degree sequence of the following graph

(Marks: 2)
Let A and B be events with

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$

Find

$P(A|B)$

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{3}}$$

$$= \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$

(Marks: 3)

Find the greatest common divisor of the following pair of integer:
72,63

Solution:

1.Divide 72 by 63:

This gives 72 = 63 * 1 + 9

2.Divide 63 by 9:

This gives 63 = 9 * 7 +0

Hence gcd (72, 63) = 9.

(Marks: 2)

Find all non isomorphic simple connected graphs with three vertices.

(Marks: 3)

How many 3-digit numbers can be formed by using each one of the digits 2,3,5,7,9 only once?

Solution:

5*4*3=60

(Marks: 3)

How many permutations of the letter of the word PANAMA can be made, if P is to be the first letter in each arrangement?

Solution:

Total letter =6
Like letters = A =3
First letter is P already selected , remaining =5
Therefore,
P (5,3) = 6

(Marks: 5) incomplete Question

A die is weighted so that the outcomes produce the following probability distribution:

Outcome	1	2	3	4	5	6
Probability	0.1	0.3	0.2	0.1	0.1	0.2

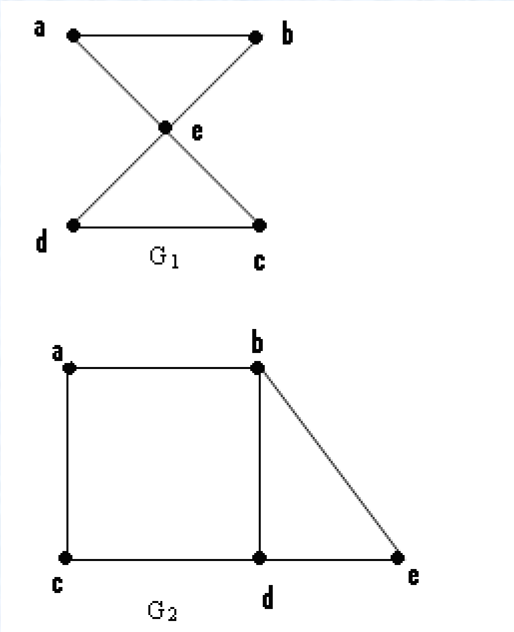
Consider the event

A= {even number} then find the following

a) P(A)
b) P (A^c)

(Marks: 5)

Determine whether the given graphs have an Euler circuit? If it does, find such a circuit, if it does not, give an argument to show why no such circuit exists.



(Marks: 5)

By using Mathematical induction prove that for all positive integers n

$1^2 + 2^2 + 3^2 + ... + n^2 = n(n+1)(2n+1)/6$

(Marks: 10)

Prove by mathematical induction that $3^{(2n-1)} + 1$ is divisible by 4 for all $n \geq 1$.

Question No: 21 (Marks: 2)

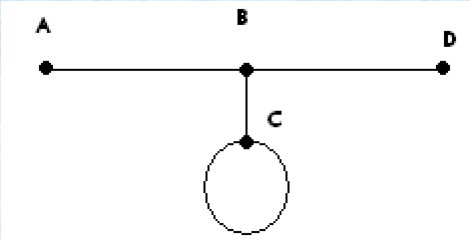
Find integers q and r so that a=bq+r , with 0≤r<b.

a=45 , b=6.

Solution:

If a =45 and b = 6 are two integers with b ≠ 0 such that the q and r are non negative integers.
a=bq+r
divides 45 by 6
this gives= 6*7 +3
divides 6 by 3
this gives = 3*2 +0
hence gcd of the (45,6) will be 3

Question No: 22 (Marks: 2)
Give the degree of each vertex in the figure (given below)



Solution:

degree of A vertex = 1
Degree of B vertex = 3
Degree of C vertex = 3
Degree of D vertex = 1
Total degree of vertices = 8
Can be prove by formula
Degree of vertices = 2. no. of edges
= 2 . 4
= 8

Question No: 23 (Marks: 2)
What is the probability of getting a number greater than 2 when a dice is tossed?

Solution:
As dice has 6 sides so possible event will be 36.
No. greater than 2 will be
3 ,4 ,5 ,6 = 4 outcomes are greater than 2

P(E) =n(E)/(n(S))

= 4/36

= 1/9 will be the possibility to get no greater than 2

Question No: 24 (Marks: 3)
How many distinguishable ways can the letters of the word HULLABALOO be arranged if words are to begin

with U and end with L

Solution:

If the words are to begin with U and end with L, then there are eight positions left to fill.
Where,
There are 3 L alike
2 O alike
2 A alike.
Therefore, the permutation becomes.

$$=\frac{8!}{3!*2!*2!}=\frac{40320}{24}=1680$$

Question No: 26 (Marks: 3)

Draw a full binary tree with seven vertices.
exact example hai Lecture 44 ki

Solution:

EXERCISE:

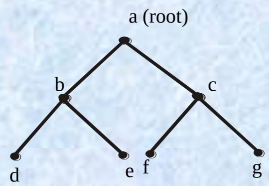
Draw a full binary tree with seven vertices.

SOLUTION:

Total vertices= 2k + 1 = 7 (by using the above theorem)

$\Rightarrow$ k = 3

Hence, total number of internal vertices (i.e. a vertex of degree greater than 1)=k=3
and total number of terminal vertices(i.e. a vertex of degree 1 in a tree) =k+1=3+1=4
Hence, a full binary tree with seven vertices is



Question No: 27 (Marks: 5)

Find n if
P(n,2) = 72

Solution:

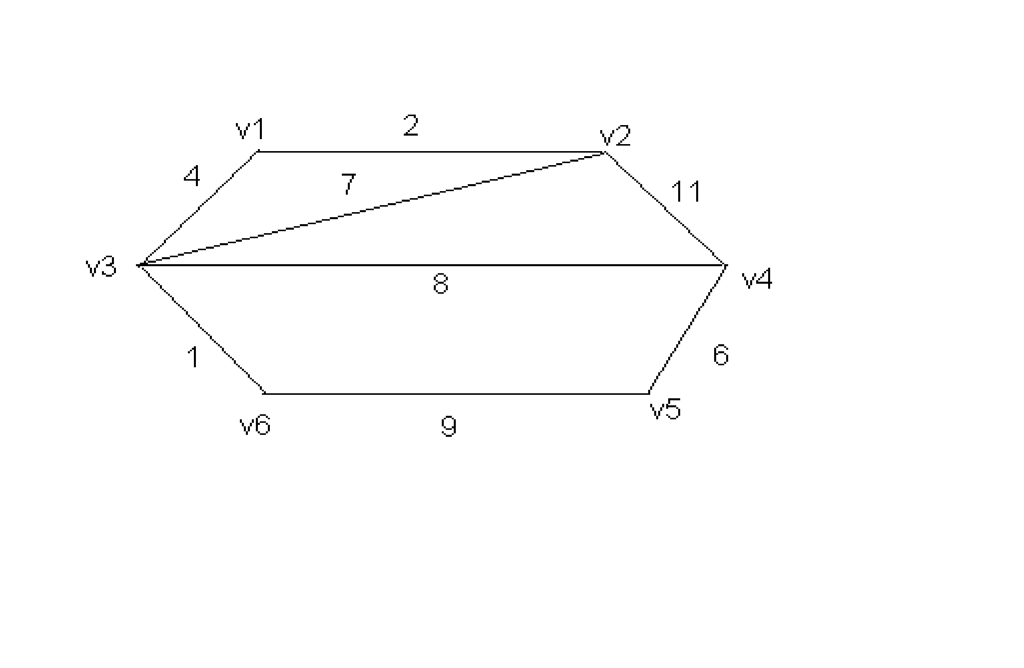
Given
P(n,2) = 72
n.(n-1)=72 by using the definition of permutation
n^2-1=72
n^2-n-72=0
n=9,-8 since n must be positive so only the acceptable value for n is 9

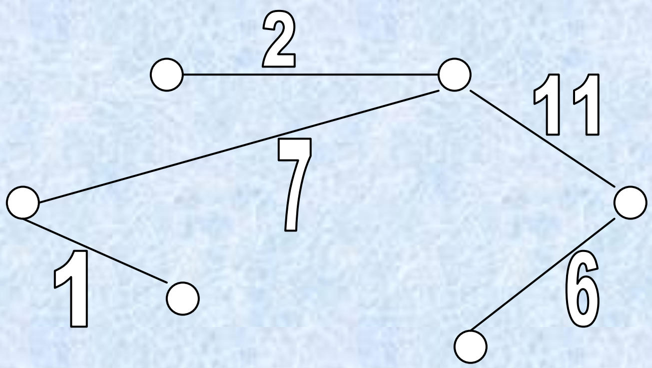
Question No: 28 (Marks: 5)

Five people are to be seated around a circular table. Two seating plans are considered as same if one is the rotation of other. How many different seating plans are possible?

Question No: 29 (Marks: 5)

Use Kruskal's Algorithm to draw the minimal spanning tree for the graph below. Indicate the order in which edges are added to form a tree.





Order of adding the edges:

{v3,v6},{v1,v2},{v4,v5},{v2,v3},{v2,v4},.

Question No: 30 (Marks: 10)

Show the sample space for tossing one penny and rolling one die.
(H = heads, T = tails) using tree diagram

Question No: 31 (Marks: 10)

,$10^{3n} + 13^{n+1}$ is divisible by 7 for all n>=1

Solution:

Let $10^{3n} + 13^{n+1}$ is divisible by 7

Basis step:
P(1) is true.now
P(1):

(1.1)

$10_{3n} + 13_{n+1}$ is divisible by 7

Since $10^{3.1}+13^{1+1} = 10^3 + 13^2$
This is divisible by 7
Hence P(1) is true.now

Inductive step:
Suppose p(k is true)
$10_{3k} + 13\ k_{+1} = 7.q$
To prove p(k+1) $10_{3n} + 13_{n+1}$ is true is divisible by 7

$10_{3k+1} + 13_{k+1+1} = 23_{4k+3}$
$= 23_{4k+3} + 2 - 2$
$= 21_{4k+3} + 2$
$= 7.3_{4k+3} + 2$
$= 7(3_{4k+3} + 2)$

$= 7.\ q$ where q is ant positive integer equal to $3_{4k+3} + 2$
So its proved that $10^{3n} + 13^{n+1}$ divisible by 7 for all n>=1